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Measuring Advances in Equality of Opportunity: The Changing Gender Gap in Educational Attainment in Canada in the Last Half Century

Gordon Anderson · Teng Wah Leo · Robert Muelhaupt

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Abstract The notion of Equality of Opportunity (EO) has pervaded much of economic and social justice policy over the last half century in conveying a sense of liberation from the circumstances that constrain an individual's ability to achieve it, and it has been a cornerstone of many gender equality programs. However unequivocal pursuit of the so called "Luck Egalitarianism" imperative has met with many critics who question why individuals who are blessed with good circumstances would wish to be "liberated" from them. This has led to a more qualified pursuit of Equal Opportunity which adds an additional proviso—that no circumstance group should be made worse off by such a policy or decentralized private initiative. Indeed observed practices, by focusing on the opportunities of the poorly endowed in circumstance, do accord with such a qualified Equal Opportunity mandate. Here it is contended that, because of the asymmetric nature of such a policy or initiative, existing empirical techniques will not fully capture the progress made toward an EO goal. Hence a new technique is introduced and employed in examining progress toward such a Qualified Equal Opportunity (QEO) Objective in the context of the educational attainments of Canadian males and females born between the 1920s and the 1970s (In the early part of that century, females did not perform as well as males educationally, and were much more constrained by their parental educational circumstance). A QEO goal is generally found to cohere with the data with females becoming less attached to their parental educational circumstance, and indeed surpassing males in their educational attainments.

G. Anderson · R. Muelhaupt
Department of Economics, University of Toronto, Toronto, ON, Canada
e-mail: anderson@chass.utoronto.ca

T. W. Leo (✉)
Department of Economics, St. Francis Xavier University, Antigonish, NS, Canada
e-mail: tleo@stfx.ca

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1 Introduction

Atkinson (2012), in his lecture in honour of Amartya Sen, avows that the aim of public policy reform is "to remedy injustice rather than characterize perfect justice". He cites the introduction to Sen (2009):

> In contrast with most modern theories of justice, which concentrate on the 'just society', this book is an attempt to investigate realization-based comparisons that focus on the advancement or retreat of justice.

in pointing out that the aim is progressive reform, rather than transcendental optimality. Here it is argued that techniques for evaluating such progressive policy reforms should also be capable of measuring the degree, and significance of such advances, or retreats. Existing techniques for evaluating the progress of public policy relating to the equality of opportunity (EO) imperative are discussed, and a new approach is introduced.

With roots in recent egalitarian political philosophy,[1] the *Equal Opportunity Imperative* sees differential outcomes as ethically acceptable when they are the consequence of individual choice and action, but not ethically acceptable when they are the consequence of circumstances beyond the individual's control. For instance, many affirmative action type policies, with equal opportunity as its objective, have been enacted to address the extent to which the distribution of individual outcomes is independent of their circumstances such as gender and race. Since an individual's circumstances have to do with the parents they were blessed with, equal opportunity policies have to address the extent to which a child's status is independent of their parents' status, in order to bring about independence between the two. In each case, the imperative can be seen to be equalizing the distributions of people's capability to achieve a favourable outcome, given their circumstance and parental status, seeking a "level playing field" with respect to their given circumstance. This paper considers the effect of societal equal opportunity aspirations in the context of the educational attainments of cohorts of Canadian men and women with respect to their gender, and parental educational circumstances, and in so doing introduces a new tool for evaluating the extent of equality of opportunity.

The equal opportunity imperative has provoked considerable empirical interest in the extent of generational mobility (or the degree to which a child's parental circumstance conditions his/her outcomes). However, systematically low mobility estimates (i.e. high correlations between parent and child outcomes, or relatively large diagonal elements in parent–child transition matrices) over many studies provided little or no support for the view that the imperative of complete independence of outcome from circumstance has been achieved.[2] One rationale for a complete mobility policy objective not being met is that, from a policy maker's perspective, other imperatives not necessarily founded upon social justice sentiments, may be in play. Piketty (2000) noted as much in his interpretation of the conservative-right wing view that, if generational mobility is low (because of the high inheritability of ability), and the distortionary costs of welfare redistributions are high,

[1] See Arneson (1989), Cohen (1989), Dworkin (1981a), Dworkin (1981b) and Dworkin (2000).

[2] See for example Bowles et al. (2005) and Corak (2004), and references therein.

it is reasonable to argue that low mobility is acceptable.[3] Friedman (2005) makes a similar point in conjecturing (with a considerable amount of supporting evidence) that economic growth has facilitated the equalizing of opportunities (amongst other improvements in social justice) in effect allowing the poor to catch up, without disadvantaging the rich.

As noted in Bowles et al. (2005) and Swift (2005), there is inevitable tension between the public desire for EO in terms of parents not influencing the outcomes of their children, and the private desire for parents to nurture their offspring. In a democratic society, public policy will ultimately be a reflection of these competing aspirations. A policy which makes at least half the inheriting groups worse off than they would have been absent policy, may not be politically viable (the parents of such groups would almost certainly not vote for such a policy) so that policy makers responding to the median voter (Downs 1957) or probabilistic voter (Coughlin 1992) mandates may wish to avoid this part of the package. Affirmative action policies are very much in this vein since they incorporate normative objectives that weigh policies in the favour of "poorly" endowed, focusing on improving the life chances of the "inherited poor", rather than diminishing the life chances of the "inherited rich".[4] In effect the policy maker has a second imperative, which is a sort of Pareto condition, wherein the lot of the poorly endowed can only be improved without diminishing the lot of the richly endowed.

With such dual mandates of equal opportunity guided by this Paretian imperative, a qualified equal opportunity program emerges with asymmetric mobility aspirations for increasing the mobility of the poorly endowed, and not increasing the mobility of the well endowed when it involves a loss of their wellbeing. The extent to which these objectives can be fulfilled is bounded by the capacity in the system to increase average child ability and outcomes. Such policies and/or decentralised initiatives can no longer be characterized as unqualified moves towards the independence of outcomes and circumstances for all groups. Rather they are equivocal moves, modifying the joint distribution of outcomes and circumstances differentially toward independence for the poor in circumstance, and independence for the rich in circumstance only if their wellbeing is not diminished. It should be noted that interest in the Paretian imperative is not guided by any normative judgement, but rather by a belief that this may well be how policies and/or initiatives are formulated in the society being studied.

Attention here is focused on gender educational equality within the Canadian context. Interest in the gender gap in educational attainment is primarily rooted in the belief that equal opportunity can best be achieved through education (Roemer 2006), and the fact that one of the preoccupations of Sen's considerable body of work on social justice is the achievement of *gender justice* (See Nussbaum 2006; Sen 1990, 1995). This could have been achieved quite swiftly by a transfer of resources from the investments in male human

[3] Indeed the pursuit of an equal opportunity goal has not been unequivocal, Anderson (1999), Cavanagh (2002), Hurley (1993) and Swift (2005) expresses some philosophical reservations, while Jencks and Tach (2006) question whether an equal opportunity imperative should require the elimination of "… all sources of economic resemblance between parents and children. Specifically … (it) … does not require that society eliminate the effects of either inherited differences in ability or inherited values regarding the importance of economic success relative to other goals". In a similar vein, Dardanoni et al. (2006) question how demanding the pursuit of equal opportunity should be in terms of the feasibility of such a pursuit.

[4] As a matter of casual empiricism, equal opportunity programs observed in "Liberal" societies do seem to be of this flavour. For example, when questioned on the widening gap between the rich and poor, the British Prime Minister responded that "… the issue is not in fact whether the very richest person ends up being richer. The issue is the poorest person is given the chance they don't otherwise have. The most important thing is to level up, not level down". Interview with the Prime Minister on BBC News Newsnight on June 4, 2001. Transcript available from http://news.bbc.co.uk/2/hi/events/newsnight/1372220.stm.

capital to investments in female human capital. Had that been so, an improvement in the achievements of females accompanied by deterioration in the achievements of males would have been observed. However it will be shown that, while male academic achievements did not deteriorate, the narrowing gender gap is characterized by an increased generational mobility of females relative to males. Furthermore, the source of this increased mobility was from the daughters of parents with lower educational attainments (which may be construed as "good" since it implies upward mobility), rather than from the daughters of parents with high educational attainments (which may be construed as a "bad" since it implies downward mobility, and the attrition of inherited ability and wellbeing). Indeed, it appears that the increased mobility of females have come about as a consequence of a reduction in the dependence of their educational outcomes on those of their mothers, especially at the lower end of the maternal educational attainment spectrum. However, increasing immobility was observed in the lowest inheritance class.

Since the 1970s, gender equity reform and policy in Canadian school system has largely been the domain of teachers and their respective associations (Coulter 1996), rather than a matter of legislation (for example Title IX of the Education Amendment Act in the US).[5] The Royal Commission on the Status of Women in Canada (2006) listed education as one of several public policy areas "particularly germane to the status of women", and many women's groups both within and outside of the ambit of the education system cited key factors contributing to women's inequality in Canada as sex-role stereotyping, the lack of strong female role-models for girls, and inadequate career counseling in schools. Beginning in the 1970s, a range of lesson plans and units were developed to assist teachers.[6] At the same time, other government agencies, institutions, and commercial publishers began producing materials for classroom use. The Ontario Institute for Studies in Education, for example, compiled *The Women's Kit* (1974), a collection of print and audio-visual materials. So began the first stage of curriculum reform.

From the perspective of identification and absolute mobility, a pure equal opportunity regime would lead to an increase in mobility for all socioeconomic/educational attainment groups, which in turn implies an *ambiguous effect on growth* in child educational outcomes. On the other hand, a qualified equal opportunity regime *requires an increase in growth* for the policy to work. Notwithstanding the above difference pertaining to outcome growth, as will become apparent, identification is primarily from the asymmetry in the attainment of mobility in terms of educational outcomes. Nonetheless, this asymmetry in mobility, where children of both high and low educational attainment parents have low measures of mobility as observed in the application, cannot be rationalised by asymmetric borrowing costs.

There are thus two main contributions in this paper. Firstly, it provides a statistically flexible measure with which to examine issues regarding mobility, or hypotheses

[5] Title IX of the Education Amendment Act of 1972 addressed discrimination with respect to gender in education. Modeled on Title IV an earlier anti-racial discrimination 1964 act, the preamble to Title IX declared that: "No person in the US shall, on the basis of sex, be excluded from participation in, be denied the benefits of, or be subject to discrimination under any educational programs or activity receiving federal financial assistance …".

[6] For example, the British Columbia Teachers' Federation (BCTF), through its Lesson Aids Service, published a variety of kits and curriculum packages with titles such as *Women in the Community*, *Famous Canadian Women*, *Early Canadian Women*, and *From Captivity to Choice: Native Women in Canadian Literature*. The Ontario Ministry of Education (1977) published a resource guide for teachers called *Sex-Role Stereotyping and Women's Studies*, which included units of study, resource lists, and teaching suggestions for teachers at all grade levels (Coulter 1996).

pertaining to joint density matrices in studies of quality of life. The method is amenable to both discrete (categorical or ordinal) and continuous variables, and remains viable in many dimensions. Secondly, it provides an identification strategy for discerning between pure equality of opportunity versus a qualified version augmented with Pareto concerns, for differing empirical strategies. These are demonstrated in a simple application within the Canadian context, examing how equality of opportunity has evolved both across parental educational groups, and gender.

In Sect. 2, the problems associated with assessing improvements in equality of opportunity using conventional methods are examined in the context of both discretely, and continuously measured indicators. The notion of qualified equality of opportunity is explored, and a new approach to examining the problem is introduced. These concepts and their measurement are employed using Statistics Canada's General Social Survey Cycle 19 (2005) to examine the closing gender gap in educational attainment that occurred in Canada[7] in Sect. 3 Finally some conclusions are drawn in Sect. 4.

2 Examining Progress Toward The Equal Opportunity Imperative

In general, the extent of equality of opportunity has been studied using one or more of three different approaches. Generational regressions have been used when the indicators of interest are continuously measured, to study the degree of dependence of child outcome on parental outcome[8] by examining the proximity to zero of the impact of parental outcomes on child outcomes, which is estimated by some form of regression technique. When the variables of interest are discrete or categorical, continuous mobility indices based upon the relative magnitudes of on and off diagonal elements of the parent–child transition matrix have been used[9] to reflect, to varying degrees, the extent to which the underlying variables are independent. With complete equality of opportunity, the columns of the transition matrix would be identical (corresponding to independence between parent and child outcomes), while with complete dependence, the transition matrix would be the identity matrix. In essence this approach formulates functions of the elements of the matrix which measure the extent to which the matrix is not supermodular (see Douglas et al. 1990; Shaked and Shanthikumar 2007). Recently an approach equivalent to comparing the distributions of the outcomes of children from different parental classes for the absence of stochastic dominance relationships between the different inheriting group distributions (LeFranc et al. 2008, 2009) has been suggested.

In the following it will be argued that changes in the coefficient on the parental outcome in a generational regression or changes in the mobility index prompted by changes in the relative magnitudes of on and off diagonal elements of a transition matrix will not adequately reflect the asymmetric nature, and hence success or failure, of equal opportunity

[7] This phenomena has also been observed in the US, see for example Blau et al. (2006), Buchmann and Diprete (2006), Dynarski (2007), Goldin et al. (2006) and Jacob (2002).

[8] Behrman and Taubman (1990), Solon (1992), Mulligan (1999), Corak and Heisz (1999), Couch and Lillard (2004), Grawe (2004), and Bratsberg et al. (2007) all being examples.

[9] Bartholemew (1982), Blanden et al. (2004), Chakravarty (1995), Dearden et al. (1997), Hart (1983), Maasoumi (1986), Prais (1955), Shorrocks (1978) and Van de Gaer et al. (2001) have all produced mobility indices many of which are discussed in Maasoumi (1996).

policies, especially if they are qualified policies. However while the stochastic dominance approach will identify the lack of equality of opportunity, unfortunately it does not yield a statistic which will indicate the degree of change or progress toward equality of opportunity. To see why this is, and understand why the evaluation of the progress that has been made, requires rethinking of current empirical approaches to equality of opportunity measurement, the structure of the equality of opportunity problem, the logic of a qualified equality of opportunity program, and its empirical implications. Broadly speaking, if there is insufficient capacity in the system to elevate the overall average ability of children, a pure equality of opportunity policy would inevitably result in the outcomes of children in some inheritance classes improving at the expense of a deterioration of outcomes of children in other inheritance classes. If the parent–child outcome relationship is monotonic and positive (as is usually the case) this means that the outcomes of children with poor parental endowments will only advance at the expense of the outcomes of richly endowed children. Such a policy may not be politically viable in a democratic system, since the interest of the median voters and their inheritors will prevail. Nonetheless, growth in capacity circumvents this problem, and facilitates a qualified equality of opportunity policy.

2.1 Qualified Equal Opportunity When Variables are Discrete

When the variable of interest is discrete (for example educational or socioeconomic status), transition matrix techniques are commonly employed. To illustrate matters in the context of a transition matrix approach (typically used when outcome measures are discrete), suppose there are four educational categories, that could be attained by both parents and children, named 1, 2, 3 and 4 which are ordered by their number so 4 is higher than 3 etc.(the model can be generalized to any and different numbers of characteristic, for both parents and children). Let the transition structure be one where the vector of parental education outcomes, [1, 2, 3, 4], transit to a vector of child outcomes, [1, 2, 3, 4]. Let the corresponding parent and child outcome probability vectors be $[p_1, p_2, p_3, p_4]$ and $[c_1, c_2, c_3, c_4]$ respectively, such that $p_k = \Pr(k)$ for parents, and $c_i = \Pr(i)$ for children. Let **J** be the matrix of joint probabilities where a typical element $j_{i,k} = \Pr(i, k)$ is the probability of observing a parent–child pairing (i, k), thus:

$$\mathbf{J} = \begin{bmatrix} j_{1,1} & j_{1,2} & j_{1,3} & j_{1,4} \\ j_{2,1} & j_{2,2} & j_{2,3} & j_{2,4} \\ j_{3,1} & j_{3,2} & j_{3,3} & j_{3,4} \\ j_{4,1} & j_{4,2} & j_{4,3} & j_{4,4} \end{bmatrix}$$

Note that $p_k = \sum_{i=1}^{4} j_{i,k}, c_i = \sum_{k=1}^{4} j_{i,k}$, and that $\sum_{i=1}^{4} ic_i = \sum_{i=1}^{4} i \sum_{k=1}^{4} j_{i,k} \leq \mu$ is a constraint on average child attainment. Let $\mathrm{P} = \mathrm{dg}(\mathbb{p})$ (dg being the diagonal operator which converts the vector into a diagonal matrix), then the conventional *transition matrix* **T** that is used for mobility indices can be written as $\mathbf{T} = \mathbf{JP}^{-1}$, whose ($i$, k)th element is $t_{i,k} = \Pr(i|k) = j_{i,k}/p_k$, and yields the child's education class vector **c** from the equation $\mathbf{c} = \mathbf{Tp}$ (Noting that $\mathbf{P}^{-1}\mathbf{p} = \mathbf{1}$, where **1** is vector of ones). In a full equal opportunity environment, parent and child outcomes will be independent, and the corresponding joint probability matrix $\mathbf{J}^I = \mathbf{cp}'$, so that the equal opportunity transition matrix $\mathbf{T}^I$ will have common columns **c**, implying the same conditional density of child outcomes in each parental category.

A pure equal opportunity program concerned solely with bringing about the independence of a child's outcome from its parents' outcome shifts any joint density $\mathbf{J}$ towards $\mathbf{J}^I$.[10] A move toward $\mathbf{J}^I$ that obeys the average child outcome constraint noted above, implies that $\sum_{i=1}^{4} i \sum_{k=1}^{4} (j^I_{i,k} - j_{i,k}) \leq 0$, and will inevitably make the children of one parental education group worse off, while making the children of another better off. To see this, first suppose the population's joint density matrix exhibits some dependence so that $\mathbf{J} \neq \mathbf{J}^I$. Consider the parental socioeconomic group denoted by the index $k = 1$. Let the nature of dependence be supermodular such that $j_{1,1} \geq j_{2,1} \geq j_{3,1} \geq j_{4,1}$. In other words, child outcomes of the lowest socioeconomic group are positively correlated with their parent's socioeconomic status, and the relationship is monotonic so that outcomes of higher endowed children weakly dominate outcomes of lower endowed children. Suppose the move towards independence shifts the attainment of children in parental group 1 towards higher attainment. Then by definition $j_{1,1} > j^I_{1,1} = c_1 p_1$, and for parental socioeconomic group 1, the following must be true,

$$\sum_{i=1}^{m} j^I_{i,1} \leq \sum_{i=1}^{m} j_{i,1} \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,1} - j_{i,1} \right) \leq 0 \tag{1}$$

where $m \in \{1, 2, 3, 4\}$. In other words, inequality (1) says that a shift towards independence leads to a stochastically dominant shift for children of parental socioeconomic group 1. However, the average child attainment constraint implies that $j_{1,k} < j^I_{1,k} = c_1 p_k$, for some $k \in \{2, 3, 4\}$, which in turn means that,

$$\sum_{i=1}^{m} j^I_{i,k} \geq \sum_{i=1}^{m} j_{i,k} \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,k} - j_{i,k} \right) \geq 0 \tag{2}$$

where $m \in \{1, 2, 3, 4\}$. Inequality (2) then says that for the higher parental group, such a policy would lead to a stochastically dominated shift. Thus a concommitant of the shift towards independence without any qualifying conditions on policy, is that the outcomes of children of higher socioeconomic status parents are necessarily diminished in order that children of low economic status are advanced. This will always happen unless there is some potential in the system for average child outcomes to grow. Indeed without any potential for growth in child outcomes, any qualified policy which at least preserves the outcomes of all children is not feasible. It is easily demonstrated in the context of this simple structure that, when there is a possibility for growth in average child outcomes, stochastically dominant shifts for the poorly parentally endowed children without concommitant stochastically dominated shifts for the parentally well endowed are feasible.[11] For this reason, progress away from a supermodular transition matrix will not be uniform, and thus not necessarily detectable with such transition matrix based mobility indices. For example, some indices computes the magnitude of the diagonal or determinant of $\mathbf{T}$, whereas movements away from supermodularity can be contrived, which do not affect values on the diagonal, or the determinant of $\mathbf{T}$. A bigger problem with these techniques is that when measured child and parent characteristics are not the same, the transition

[10] This is the same interpretation as that in Van de Gaer et al. (2001) since with the independence structure, the probability of attaining an outcome is the same for all children regardless of their parent's educational status. The sole difference being the emphasis on joint density here versus the transition matrix in Van de Gaer et al. (2001).

[11] Since this will be demonstrated in the following continuous case, it will not be reported here, but is available from the authors on request.

approach is generally not viable. All of which makes rendering inferences about the progress of equality of opportunity programs very difficult.

2.2 Qualified Equal Opportunity When Variables are Continuous

When the variable of interest is continuous (for example incomes), equality of opportunity has frequently been examined via the regression coefficient (β) of a child's characteristic when adult ($y \in Y$) on the corresponding parental characteristic ($x \in X$).

$$y = \alpha + \beta x + \gamma x^2 + \epsilon$$

where ϵ is the population error term. The literature building upon Becker and Tomes (1979) created a rich class of models highlighting the forces that determined the value of β (with γ set to 0), which is interpreted as a mobility index, where it inferred mobility (equal opportunity) as $\beta \to 0$, and immobility (unequal opportunity) as $\beta \to 1$. Since Atkinson (1983) there has been interest in the nonlinearity of generational income elasticity ($\gamma < 0$) or asymmetry of mobility, largely stimulated by the Becker and Tomes (1986) conjecture that parent–child outcome relationships are concave due to asymmetries in borrowing constraints. It is worth examining whether qualified equal opportunity policies could affect the structure of the parent–child relationship, within the context of a generational regression.

In this continuous paradigm, the policy maker's dilemma can be illustrated as follows. Suppose an initial pre-policy state, with parental outcome $x \in X$ distributed with density $f(x)$, and c.d.f. $F(x)$, with $\mathbf{E}(x) = \mu$, $\mathbf{V}(x) = \sigma^2$, and where child outcome when adult is given by:

$$y = (1 - \xi)x + \xi e \tag{3}$$

where $0 \leq \xi \leq 1$, and e is distributed as $g(e)$, where $g(x) = f(x)$ for all x, and $h(x, e) = f(x)g(e)$ (That is to say x and e are identically but independently distributed). So that Immobility (Unequal Opportunity) implies $\xi = 0$, when child outcomes are entirely determined by their parental circumstances, and Mobility (Equal Opportunity) implies $\xi = 1$ where child outcomes are entirely determined by luck. Then $\mathbf{E}(y) = \mu$ (implying that average ability is constant, so that there is no growth between generations in this society), and $\mathbf{V}(y) = (1 + 2\xi(\xi - 1))\,\sigma^2$ for all $\xi \in [0,1]$. For convenience, let $f(x) \sim N(\mu, \sigma^2)$, and note that:

$$f(y|x) \sim N\left((1 - \xi)x + \xi\mu, \xi^2\sigma^2\right)$$

for $\xi > 0$, which accords with the constraint that $\mathbf{E}(y) \leq \mu$. Thus in the pre-policy state,

$$\frac{\partial \mathbf{E}(y|x)}{\partial x} = (1 - \xi) \tag{4}$$

$$\frac{\partial \mathbf{V}(y|x)}{\partial x} = 0 \tag{5}$$

Implying that the intergenerational relationship is linear, and constant across socioeconomic groups, and the relationship is homoskedastic, much like the assumptions underlying the generational regressions commonly found in the literature.

Let $\Phi(.)$ and $\phi(.)$ denote the standard normal c.d.f. and p.d.f. respectively, then for all $\xi < 1$, children with parental outcome x^* have a distribution of outcomes that first order dominate those of children with parental outcome x^{**} when $x^* > x^{**}$, since for all y,

$$\begin{aligned} F(y|x^*) &= \Phi\left(\frac{Y-((1-\xi)x^*+\xi\mu)}{\xi\sigma}\right) \\ &\leq \Phi\left(\frac{Y-((1-\xi)x^{**}+\xi\mu)}{\xi\sigma}\right) \\ &= F(y|x^{**}) \end{aligned}$$

with strict inequality holding for some Y. Essentially well endowed children are better off than poorly endowed children except under perfect mobility ($\xi = 1$). This is what in effect motivates the stochastic dominance approach to examining equality of opportunity (LeFranc et al. 2008, 2009) since it seeks to see whether or not the above inequality holds for any pairs x^*, and x^{**}.

Pure EO policies attempt to increase ξ uniformly across $x \in X$. Consider the marginal effect of an increase in ξ on the probability that a child's outcome is less than Y given parental outcome x^*:

$$\begin{aligned} \frac{\partial \Pr(y<Y|x^*)}{\partial\xi} &= \frac{\partial F(Y|x^*)}{\partial\xi} \\ &= \frac{\partial\Phi\left(\frac{Y-\mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)}{\partial\xi} \\ &= \phi\left(\frac{Y-\mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)\frac{\partial\frac{Y-\mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}}{\partial\xi} \\ &= \phi\left(\frac{Y-\mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)\left(\frac{x^*-Y}{\lambda^2\sigma}\right) \end{aligned}$$

That is the marginal effect on child outcome of this policy is positive for $x^* > Y$, and negative for $x^* < Y$. Thus, $\frac{\partial \Pr(y<Y|x^*)}{\partial\xi} \leq 0$ as $x^* \leq Y$, and $\frac{\partial \Pr(y<Y|x^*)}{\partial\xi} > 0$ as $x^* > Y$. Hence, $F_{post}(y|x^*) - F_{pre}(y|x^*) \geq 0$ for $y \leq x^*$, and $F_{post}(y|x^*) - F_{pre}(y|x^*) < 0$ for $y > x^*$, so that the pre- and post-policy change cumulative densities, F_{pre} and F_{post} cross just once at x^*. Here note that both pre- and post-policy outcome distributions are normally distributed so:

$$\int_{\infty}^{x^*}\left(F_{pre}(y|x^*)-F_{post}(y|x^*)\right)dy \gtreqless \int_{x^*}^{\infty}\left(F_{pre}(y|x^*)-F_{post}(y|x^*)\right)dy \quad \text{as} \quad x^* \gtreqless \mathbf{E}(Y)$$

which means that pre-policy outcomes second order dominate post policy outcomes for high (above average) parental groups so that average child outcomes diminish for these groups, whereas post-policy outcomes second order dominate pre-policy outcomes in the counter cumulative density[12] sense for low (below average) parental groups, so that average child outcomes increase for these groups. If, in this very stylized symmetric world,

[12] Second order dominance of the counter cumulative density

$$\int_{x}^{\infty}\left(F_{pre}(z)-F_{post}(z)\right)dz \geq 0 \quad \forall x$$

with strict inequality holding somewhere, is a sufficient condition for $\mathbf{E}_{pre}(Y) = \mathbf{E}_{post}(Y)$ (Anderson 2004; Levy and Wiener 1998).

parents vote selfishly in the interests of their children, probabilistic or median voter models would predict a tie between pro EO policy (below median parents), and status quo (above median parents) voters (the median voter would be indifferent between the two states), and the policy would face considerable electoral uncertainty.

Consider now a qualified equal opportunity policy where the policy maker is inclined to increase ξ more for children from lower socioeconomic status families, and less for those from higher socioeconomic status families (and most importantly it has the growth capacity to do so), so that ξ now becomes a linear decreasing function of x with $\xi'(x) < 0$, $0 < \xi(x) \leq 1$ ($\xi''(x) = 0$ is assumed for simplicity). Denote the density of the child's outcome as $f^q(.)$, and the distribution as $F^q(.)$. It follows that:

$$f^q(y|x) \sim N\Big((1 - \xi(x))x + \xi(x)\mu, \xi(x)^2\sigma^2\Big)$$

It is readily seen that $\mathbf{E}(Y)$ under this qualified conditional distribution is greater than $\mathbf{E}(Y)$ under the unqualified conditional distribution, so that average child quality has increased. Obviously the policymaker has the wherewithal to do this, otherwise the policy is not feasible. In the post-policy state, among families affected by the qualified equal opportunity policy,

$$\frac{\partial \mathbf{E}(y|x)}{\partial x} = 1 - \xi(x) + \xi'(x)(\mu - x) \tag{6}$$

$$\frac{\partial^2 \mathbf{E}(y|x)}{\partial x^2} = -2\xi'(x) + \xi''(x)(\mu - x) = -2\xi'(x) > 0 \tag{7}$$

First note that the parent–child relationship is no longer constant across socioeconomic groups, and that $\mathbf{E}(y|x)$ is convex in x compared to the linear relationship of Eq. (4). In addition,

$$\frac{\partial \mathbf{V}(y|x)}{\partial x} = 2\xi(x)\xi'(x)\sigma^2 < 0 \tag{8}$$

implying heteroskedasticity that diminishes with x, instead of homoskedasticity of Eq. (5). This implies an increased variance for the poorly endowed, which is the primary measure of the extent to which children of low outcome parents have been released from their circumstance. In terms of voting behaviour the impact on the status quo group has been lessened, and that on the pre-policy group increased so that, within the context of a probabilistic voting model, the electoral uncertainty regarding the policy has diminished. This would suggest a quantile regression approach (Koenker 2005). Further, due attention should still be paid to heteroskedasticity in the error process that this structure engenders[13] even in a quantile regression approach, which has never, to the authors knowledge, been applied in this context.

To restate the key insights, the analysis suggests that whatever the initial generational regression relationship, a qualified equal opportunity program would (1) convexify (or reduce the concavity of) the parent−child dependence structure, and (2) make increasingly negative, the relationship between the conditional error heteroskedasticity and parental socioeconomic status.

[13] Unfortunately due to the categorical nature of the data employed here this approach cannot be explored in what follows.

2.3 The Overlap Technique

The technique proposed here measures how close the actual joint density of parent–child outcomes is to one which reflects either independence (EO) or qualified independence (QEO) in parent–child outcomes. It does so by measuring the degree of overlap of the two distributions.[14] The principal benefits of the measure is in its ease of application, its statistical properties (it is asymptotically Normal when based upon a random sample, consequently permitting inference), and its amenability to examining both continuous and discrete variables, and mixtures thereof in multiple dimensions (see Anderson et al. (2010), (2012) and Anderson and Hachem (2012) for details). Specifically, for the empirical application here, the focus is on,

$$\mathbf{OV} = \sum_{i \in I} \sum_{k \in K} \min\left\{ j^o_{i,k}, j^e_{i,k} \right\} \tag{9}$$

where $j^o_{i,k}$ is the typical element of $\mathbf{J}^o$ or observed joint density matrix, and $j^e_{i,k}$ the typical element of $\mathbf{J}^e$ in the theoretical joint density matrix. Further, the technique is amenable to examining not only the independence hypothesis, but any conceivable hypothesis. Intuitively, the overlap measure is depicted below in Fig. 1, where the distributions f and g correspond to the observed, and theoretical distributions respectively. Note that the more f and g coincide, the more will the overlap measure tend toward 1, while the more they diverge the more it will tend toward 0.

For instance, for examining the independence hypothesis, the overlap measure would then be

$$\mathbf{OV}^{Idp} = \sum_{i \in I} \sum_{k \in K} \min\left\{ j^o_{i,k}, c_i p_k \right\} \tag{10}$$

Further, the measure can be focused on any specific dimension so as to dissect the analysis further. For example in the case of a qualified equal opportunity program, the key to identifying if it is benefitting a particular segment of the populace, is through calculating the overlap measure for each such group.

$$\mathbf{OV}^{Idp}_k = \sum_{i \in I} \min\left\{ \frac{j^o_{i,k}}{p_k}, c_i \right\} \tag{11}$$

3 Changes in Gender Gaps, Parental Circumstances and Educational Attainment in Canada

One profound change in the latter part of the twentieth century which exemplifies the equal opportunity mandate was the emancipation of women from the household, and the declining significance of gender concerns in labour markets, thereby raising the incentives for educational pursuits amongst women (Blau et al. 2006). The advancements made in

[14] The Overlap Measure proposed in this paper can be adapted to the three conceptions of intergenerational mobility, namely movements across groups, an index of equality of opportunity, and an index of life chances, suggested by Van de Gaer et al. (2001), since each transition matrix has an implied structure on the joint density matrix, which the empirical joint density can be measured against. Further, the third mobility measure for Markov chains proposed by Van de Gaer et al. (2001) is related to the Overlap measure in the sense that it measures the *complement* to the overlapping region of the conditional probabilities.

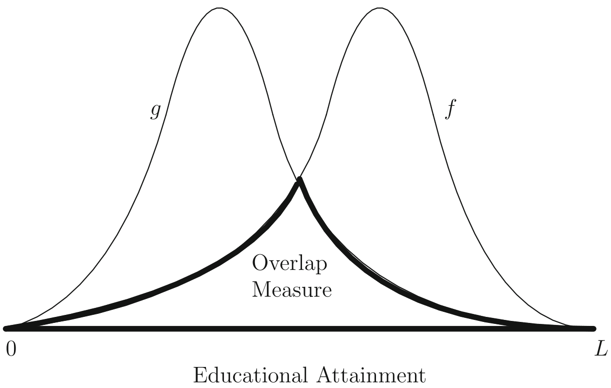

Fig. 1 Overlap between densities *f* and *g*, *OV*

women's status, and wellbeing was possible due to the introduction of the pill, abortion rights and legislation against gender discrimination in the workplace (Goldin and Katz 2002; Pezzini 2005; Siow 2002). One dimension in which this found expression is in the narrowing of the gender gap in academic achievement (Dynarski 2007). To study this phenomenon in light of the hypothesis that equal opportunity policies are qualified in nature, the educational achievements of successive cohorts of Canadian women conditional on their parents' are compared both across cohorts, and against their male counterparts. In addition, a priori under a qualified equal opportunity policy focused on enhancing the wellbeing of families of lower socioeconomic status as measured by educational attainment, we should see improvements in the outcomes of their children, regardless of their gender, without affecting the highest attainment groups. Amalgamating these two concerns, the greatest improvements in equality of opportunity should be observed in women from the modal, and lower socioeconomic status families in later cohorts.

3.1 Summary of Data

The data for the empirical analysis on academic achievements of children and their parents in Canada are drawn from Statistics Canada's *General Social Survey Cycle 19* (2005). Educational attainment is indexed from 1 to 5 as follows: 1 for some secondary/elementary/no education; 2 for high school diploma; 3 for some university; 4 for Diploma/Certificate in a Trade/Technical skill, and 5 for a university degree. This categorization is for all individuals above the age of 25, including both parents and their children.

Table 1 summarizes the proportion of individuals in each educational attainment category, and the corresponding proportion of observations with their parents in those categories by the individual's gender and cohort (decade in which they were born). Although the analysis was performed for all cohorts, the following results reports, and focuses only on three cohorts, the 70s, 50s, and the 30s and prior cohorts.[15] Notice first that amongst individuals born in the 1930s and earlier, the upper attainment levels (in terms of proportions) are dominated by males, but this changes in favour of females in later cohorts, corresponding with the increased female labour force participation in the post World War

[15] The discussion that follows extends to the full dataset, and the results in its entirety are available from the authors upon request.

Table 1 Summary statistics by gender and cohort

Decade	Gender	No. of obs.	Variable	Some/No school	High school	Some college	Technical education	University
70s	Male	895	Own	0.07	0.14	0.15	0.32	0.31
			Father's	0.27	0.32	0.07	0.12	0.23
			Mother's	0.20	0.41	0.05	0.13	0.21
	Female	1,187	Own	0.05	0.12	0.14	0.32	0.37
			Father's	0.30	0.29	0.06	0.15	0.20
			Mother's	0.27	0.36	0.08	0.16	0.13
50s	Male	995	Own	0.10	0.18	0.16	0.25	0.30
			Father's	0.57	0.22	0.04	0.04	0.13
			Mother's	0.48	0.35	0.03	0.05	0.09
	Female	1,201	Own	0.07	0.17	0.16	0.32	0.27
			Father's	0.56	0.24	0.03	0.04	0.12
			Mother's	0.54	0.28	0.04	0.08	0.07
≤30s	Male	569	Own	0.31	0.14	0.13	0.14	0.28
			Father's	0.72	0.15	0.05	0.02	0.06
			Mother's	0.67	0.19	0.04	0.04	0.05
	Female	887	Own	0.37	0.19	0.12	0.19	0.14
			Father's	0.72	0.14	0.03	0.04	0.07
			Mother's	0.72	0.16	0.02	0.05	0.05

II decades noted previously. Nonetheless, on the aggregate, there is growth in the average attainment levels amongst the children of each cohort compared to the previous cohorts. As already noted, an EO policy without growth, such as a pure equal opportunity policy would engender improvements amongst children of lower socioeconomic status, but a fall in outcomes amongst children of higher socioeconomic status. Finally, notice the shift in the modal educational attainment amongst parents from "some/no school" in the pre-30s cohorts to "high school" by the arrival of the 70s cohort. This has implications regarding the modal group that may be driving the asymmetry in intergenerational mobility through the five decades as alluded to in the above discussion.

Since a QEO program requires both stochastic dominance criteria, and overall growth in achievement, these conditions are highlighted in Tables 2 and 3. Firstly, Table 2 examines whether through the five decades, children of each parental educational group were made unambiguously worse off or otherwise by employing stochastic dominance tests as prescribed by Linton et al. (2005), and Dardanoni and Forcina (1998). With the exception of a few 70s versus 50s comparisons amongst parents with some college and technical education where no dominance relationship was registered, each parental educational group for each cohort stochastically dominated its preceding cohort unanimously, so improvements in educational outcomes were more or less ubiquitous as expected.

With regard to gender equality, Table 3 examines the dominance relationship between the genders, and shows the improvement in outcomes of females relative to males that have been made over the period. For the 70s cohort, female child educational outcomes first order stochastically dominate their male counterparts, a complete reversal from the status quo four decades previously. Nonetheless, this analysis does not provide us with any evidence regarding changes in the structure of the parent–child connection, which is the primary purpose in any equal opportunity program, qualified or otherwise. In other words, the question remains as to whether there has been gains in intergenerational mobility for children of lower socioeconomic status, and particularly amongst female children. This latter concern will be examined using both the Stochastic Dominance technique of LeFranc et al. (2008, 2009), the Overlap Measure and, for illustrative purposes, a regression analysis.

Following LeFranc et al. (2008, 2009), Table 4 reports the stochastic dominance comparisons of outcome distributions of children of different parental groups for each of the three cohorts. For the 70s and 50s cohorts, with the exception of the technical/college comparison for which no dominance relationship was established, the distributions of children with higher parental circumstance invariably dominates significantly that of children with lower parental circumstance. This implies an absence of equality of opportunity for those two cohorts. For the 30s cohort, there were three reversals (lower inheritance groups dominating higher inheritance groups) which is somewhat less conclusive. In any event, this does not provide any evidence of a trend toward or away from an equality of opportunity outcome structure, so attention turns to the overlap measure.

3.2 Examining Qualified Equal Opportunity Using the Overlap Measure

The qualified equal opportunity hypothesis suggests that the conditional density of child attainment for lower socioeconomic groups should be a closer match to the marginal density of child attainment relative to the children from higher socioeconomic status groups, since a qualified policy would leave the latter group relatively untouched. Equation (11) above provides a test that will be performed, which intuitively measures the degree of overlap between two densities for each parental socioeconomic status/educational

Table 2 Stochastic dominance tests between cohorts by parental attainment

Parental education	Some/No school		High school		Some college		Technical education		University	
Hypothesis	70s $\succ^i$ 50s	50s $\succ^i$ 70s	70s $\succ^i$ 50s	50s $\succ^i$ 70s	70s $\succ^i$ 50s	50s $\succ^i$ 70s	70s $\succ^i$ 50s	50s $\succ^i$ 70s	70s $\succ^i$ 50s	50s $\succ^i$ 70s
Statistic (father)	0.19	1.27	0.19	1.99	0.12	0.33	0.23	0.38	0.29	1.21
$\Pr(Z \leq z)$	[0.07]	[0.96]	[0.07]	[1.00]					[0.15]	[0.95]
Result	70s $\succ^1$ 50s		70s $\succ^1$ 50s		No dominance		No dominance		70s $\succ^1$ 50s	
Statistic (mother)	0.00	3.03	0.35	1.14	1.46	0.31	0.35	0.48	0.12	1.33
$\Pr(Z \leq z)$	[0.00]	[1.00]	[0.21]	[0.93]	[0.99]	[0.18]			[0.03]	[0.97]
Result	70s $\succ^1$ 50s		70s $\succ^1$ 50s		50s $\succ^1$ 70s		No dominance		70s $\succ^1$ 50s	
Hypothesis	70s $\succ^i$ 30s	30s $\succ^i$ 70s	70s $\succ^i$ 30s	30s $\succ^i$ 70s	70s $\succ^i$ 30s	30s $\succ^i$ 70s	70s $\succ^i$ 30s	30s $\succ^i$ 70s	70s $\succ^i$ 30s	30s $\succ^i$ 70s
Statistic (father)	0.00	19.07	0.30	4.46	0.00	4.09	0.13	1.66	0.00	2.83
$\Pr(Z \leq z)$	0.00	1.00	0.16	1.00	0.00	1.00	0.03	1.00	0.00	1.00
Result	70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s	
Statistic (mother)	0.00	19.03	0.73	4.67	0.00	3.76	0.00	3.40	0.00	3.52
$\Pr(Z \leq z)$	[0.00]	[1.00]	[0.66]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]
Result	70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s		70s $\succ^1$ 30s	
Hypothesis	50s $\succ^i$ 30s	30s $\succ^i$ 50s	50s $\succ^i$ 30s	30s $\succ^i$ 50s	50s $\succ^i$ 30s	30s $\succ^i$ 50s	50s $\succ^i$ 30s	30s $\succ^i$ 50s	50s $\succ^i$ 30s	30s $\succ^i$ 50s
Statistic (father)	0.00	21.89	0.14	2.80	0.00	3.53	0.21	1.39	0.00	2.07
$\Pr(Z \leq z)$	0.00	1.00	0.04	1.00	0.00	1.00	0.09	0.98	0.00	1.00
Result	50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s	
Statistic (mother)	0.00	20.22	0.47	3.75	0.00	5.44	0.00	3.21	0.00	2.31
$\Pr(Z \leq z)$	[0.00]	[1.00]	[0.36]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]
Result	50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s		50s $\succ^1$ 30s	

$\Pr(Z \leq z)$ in brackets

$\succ^i$ denotes stochastic dominance of order i tested

***, ** Corresponds to rejection of hypothesis at 1 and 5 % level of significance, respectively

Table 3 Males versus Females stochastic dominance by cohort

Hypothesis	70s cohort		50s cohort		$\leq$30s cohort	
	Males $\succ^i$ Females	Females $\succ^i$ Males	Males $\succ^i$ Females	Females $\succ^i$ Males	Males $\succ^i$ Females	Females $\succ^i$ Males
Statistic	1.40	−0.56	0.83	7.44	0.00	7.60
$\Pr(Z \leq z)$	[0.98]	[0.00]		[1.0000]	[0.00]	[1.00]
Result	Females $\succ^1$ Males**		Males $\succ^2$ Females***		Males $\succ^1$ Females***	

$\Pr(Z \leq z)$ in brackets

$\succ^i$ denotes stochastic dominance of order i tested. For the 50s cohort, the 1st order stochastic dominance test was indeterminate, so that the 2nd order stochastic dominance test was performed

***, **, * Corresponds to the result obtained at the 1, 5 and 10 % level of significance respectively

attainment group. In obtaining the measure, all observations were weighted by their individual weights. If the child's educational outcome and parental circumstances are independent, the Overlap measure will record values close to 1 corresponding to equality of opportunity for that group. To the extent that they are not independent, the statistic will record a value substantially <1 reflecting the greater attachment of a child's outcome to its parents. The results of this measure for each parental attainment outcome by the gender of the children are reported in Table 5.

What is immediately striking from Table 5 is the ubiquitous trend over the decades toward greater equality of opportunity amongst children of both genders from families with parents who have high school, some college, and technical education over the decades, as reflected in the closer proximity of the overlap measure to 1. Improvements in EO amongst children of parents of university degrees are somewhat more attenuated, with the overlap measure remaining significantly lower than for children in the other educational groups. The gains are particularly impressive for children of parents with education at or above a high school diploma when compared to the low levels of overlap amongst the generation in the oldest cohort from the 30s. Due to the asymptotic normality of the measures, the significance of the improvements can be determined by a standard normal test of their differences. The value of the statistic informs us of the magnitude of improvement, while the sign tells us if mobility improved (if positive) or fell (if negative). The magnitude of these gains are reported in Table 5 below the corresponding overlap measures.

Amongst male children, those from families with parents with education at or greater than a high school diploma enjoyed significant gains up to the 50s. However, there was an apparent slow down in the gains in EO amongst the 70s cohort, with some evidence of children of high school diploma parents suffering a fall. This gain is also apparent amongst female children, with the evidence here being far stronger, and more consistent through to the 70s for women of parents with education at and beyond high school. In all of this, children of parents who had not completed their high school diploma, regardless of gender, saw a consistent decline in EO, suggesting that EO policies were not effective in elevating the state of these most disadvantaged children. However it should be noted that the levels of EO, as evident from the proximity of the overlap measure to 1, has been high amongst these disadvantaged children. In other words, their social mobility has always been closer to independence, in stark contrast to children of parents with university education.

Table 6 reports comparisons by gender, by differences in the overlap measure of female children of mothers against male children of fathers (the results for other comparisons are

Table 4 Stochastic dominance between children of parents of differing attainment by cohort

Father's education		High school		College		Technical education		University	
		70s cohort							
No school	Hypothesis	No sch. $\succ^i$ High	High $\succ^i$ No sch.	No sch. $\succ^i$ Coll.	Coll. $\succ^i$ No sch.	No sch. $\succ^i$ Tech.	Tech. $\succ^i$ No sch.	No sch. $\succ^i$ Uni.	Uni. $\succ^i$ No sch.
	Statistic	1.77	−0.95	2.22	−1.03	2.57	−1.16	6.15	−1.66
	P value	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
	Result	High $\succ^1$ No sch.		College $\succ^1$ No sch.		Tech. $\succ^1$ No sch.		Uni. $\succ^1$ No sch	
High school	Hypothesis			High $\succ^i$ Coll.	Coll. $\succ^i$ High	High $\succ^i$ Tech.	Tech. $\succ^i$ High	High $\succ^i$ Uni.	Uni. $\succ^i$ High
	Statistic			1.68	−0.45	1.86	−0.40	4.49	−0.79
	P value			[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
	Result			College $\succ^1$ High		Tech. $\succ^1$ High		Uni. $\succ^1$ High	
College	Hypothesis					College $\succ^i$ Tech.	Tech. $\succ^i$ College	College $\succ^i$ Uni.	Uni. $\succ^i$ College
	Statistic					0.03	0.78	1.78	−0.05
	P Value							[1.00]	[0.00]
	Result					No dominance		Uni. $\succ^1$ College	
Tech. education	Hypothesis							Tech. $\succ^1$ Uni.	Uni. $\succ^1$ Tech.
	Statistic							2.87	−0.27
	P value							[1.00]	[0.00]
	Result							Uni. $\succ^1$ Tech.	
		50s Cohort							
No school	Hypothesis	No sch. $\succ^i$ High	High $\succ^i$ No sch.	No sch. $\succ^i$ Coll.	Coll. $\succ^i$ No sch.	No sch. $\succ^i$ Tech.	Tech. $\succ^i$ No sch.	No sch. $\succ^i$ Uni.	Uni. $\succ^i$ No sch.
	Statistic	1.91	−1.07	1.84	−0.80	2.16	−0.71	5.72	−1.09
	P value	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
	Result	High $\succ^1$ No sch.		College $\succ^1$ No sch.		Tech. $\succ^1$ No sch.		Uni. $\succ^1$ No sch.	
High school	Hypothesis			High $\succ^i$ Coll.	Coll. $\succ^i$ High	High $\succ^i$ Tech.	Tech. $\succ^i$ High	High $\succ^i$ Uni.	Uni. $\succ^i$ High
	Statistic			1.67	−0.28	1.95	−0.15	3.73	−0.23
	P value			[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
	Result			College $\succ^1$ High		Tech. $\succ^1$ High		Uni. $\succ^1$ High	

Table 4 continued

Father's education		High school		College		Technical education		University	
College	Hypothesis					Coll. $\succ^i$ Tech.	Tech. $\succ^i$ Coll.	Coll. $\succ^i$ Uni.	Uni. $\succ^i$ Coll.
	Statistic					0.10	0.47	1.71	0.14
	P value							[1.00]	[0.04]
	Result					No dominance		Uni. $\succ^1$ College	
Tech. education	Hypothesis							Tech. $\succ^i$ Uni.	Uni. $\succ^i$ Tech.
	Statistic							2.19	−0.01
	P value							[1.00]	[0.00]
	Result							Uni. $\succ^1$ Tech.	
		30s cohort							
No school	Hypothesis	No sch. $\succ^i$ High	High $\succ^i$ No sch.	No Sch. $\succ^i$ Coll.	Coll. $\succ^i$ No sch.	No sch. $\succ^i$ Tech.	Tech. $\succ^i$ No sch.	No Sch. $\succ^i$ Uni.	Uni. $\succ^i$ No sch.
	Statistic	3.72	−3.72	2.04	−1.69	2.67	−2.03	4.07	−2.84
	P value	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
	Result	High $\succ^1$ No sch.		Coll. $\succ^1$ No sch.		Tech. $\succ^1$ No sch.		Uni. $\succ^1$ No sch.	
High school	Hypothesis			High $\succ^i$ Coll.	Coll. $\succ^i$ High	High $\succ^i$ Tech.	Tech. $\succ^i$ High	High $\succ^i$ Uni.	Uni. $\succ^i$ High
	Statistic			0.17	1.31	1.23	−0.21	1.91	−0.28
	P value				[0.95]	[0.95]	[0.00]	[1.00]	[0.00]
	Result			High $\succ^2$ Coll.		Tech. $\succ^1$ High		Uni. $\succ^1$ High	
College	Hypothesis					Coll. $\succ^i$ Tech.	Tech. $\succ^i$ Coll.	Coll. $\succ^i$ Uni.	Uni. $\succ^i$ Coll.
	Statistic					0.00	5.01	1.14	−0.34
	P value						[0.99]	[0.93]	[0.00]
	Result					Coll. $\succ^2$ Tech.		Uni. $\succ^1$ Coll.	

Table 4 continued

Father's education		High school		College		Technical education		University	
Tech. education	Hypothesis							Tech. $\succ^1$ Uni.	Uni. $\succ^1$ Tech.
	Statistic							0.00	3.03
	P Value								[0.99]
	Result							Tech. $\succ^2$ Uni.	

$\Pr(Z \leq z)$ in brackets

$\succ^i$ denotes stochastic dominance of order *i* tested

***, ** Corresponds to rejection of hypothesis at 1 and 5 % level of significance respectively

Table 5 Qualified mobility indices by parental attainment class, cohort and gender

	Mother's attainment					Father's attainment				
	Drop out	High school	Some college	Technical education	University	Drop out	High school	Some college	Technical education	University
	Panel A: Male child									
70s cohort	0.878	0.918	0.905	0.934	0.777	0.829	0.928	0.859	0.835	0.738
	(0.011)	(0.009)	(0.010)	(0.008)	(0.014)	(0.013)	(0.009)	(0.012)	(0.012)	(0.015)
50s cohort	0.891	0.952	0.779	0.773	0.726	0.915	0.936	0.775	0.842	0.701
	(0.010)	(0.007)	(0.013)	(0.013)	(0.014)	(0.009)	(0.008)	(0.013)	(0.012)	(0.015)
≤30s cohort	0.914	0.827	0.741	0.624	0.657	0.918	0.799	0.760	0.527	0.617
	(0.012)	(0.016)	(0.018)	(0.020)	(0.020)	(0.012)	(0.017)	(0.018)	(0.021)	(0.020)
70s–50s	−0.887	−3.025**	7.630***	10.243***	2.541**	−5.609***	−0.645	4.788***	−0.447	1.787**
70s– ≤30s	−2.208**	4.938***	7.874***	14.105***	4.929***	−5.232***	6.820***	4.647***	12.617***	4.822***
50s– ≤30s	−1.459*	7.252***	1.720**	6.142***	2.838**	−0.203	7.364***	0.667	13.160***	3.367***
	Panel B: Female child									
70s cohort	0.838	0.970	0.807	0.903	0.742	0.850	0.955	0.878	0.919	0.790
	(0.012)	(0.005)	(0.012)	(0.009)	(0.013)	(0.010)	(0.006)	(0.010)	(0.008)	(0.012)
50s cohort	0.919	0.942	0.663	0.854	0.774	0.914	0.894	0.866	0.811	0.674
	(0.008)	(0.007)	(0.014)	(0.010)	(0.012)	(0.008)	(0.009)	(0.010)	(0.011)	(0.014)
≤30s cohort	0.913	0.752	0.655	0.627	0.599	0.904	0.780	0.83	0.603	0.628
	(0.010)	(0.015)	(0.016)	(0.016)	(0.017)	(0.010)	(0.014)	(0.013)	(0.016)	(0.016)
70s–50s	−6.067***	3.316***	8.094***	3.670***	−1.830**	−4.868***	5.663***	0.830	7.857***	6.456***
70s– ≤30s	−5.230***	14.219***	7.735***	15.022***	6.862***	−3.821***	11.533***	3.013**	17.309***	8.050***
50s– ≤30s	0.478	11.897***	0.371	11.846***	8.561***	0.733	6.913***	2.268**	10.388***	2.160**

Standard errors are in Parenthesis. All overlap measures reveal that there is significance difference vis-à-vis the marginal density under complete independence

$\Pr(Z \leq z)$ in brackets

The Z statistic is reported for the comparisons of overlap measure across the cohorts. A positive(negative) statistic implies that mobility rose(fell), and the magnitude of the statistic is a reflection of the degree of improvement in intergenerational mobility

***, **, * Corresponds to the statistical significance at the 1, 5 and 10 % levels, respectively

Table 6 $H_0 : \mathbf{OV}^{Idp}_{daughters} - \mathbf{OV}^{Idp}_{sons} \geq 0$ versus $H_1 : \mathbf{OV}^{Idp}_{daughters} - \mathbf{OV}^{Idp}_{sons} < 0$

	Parental attainment				
	Drop out	High school	Some college	Technical education	University
70s cohort	0.576	4.232	−3.226***	4.551	0.183
	[0.718]	[1.000]	[0.001]	[1.000]	[0.573]
50s cohort	0.319	0.676	−5.921***	0.796	3.843
	[0.625]	[0.751]	[0.000]	[0.787]	[1.000]
≤30s Cohort	−0.340	−2.108**	−4.399***	3.772	−0.686
	[0.367]	[0.018]	[0.000]	[1.000]	[0.247]

The null hypothesis is for the attainment density of "Daughters of Mothers" being closer to independence than "Sons of Fathers"

$\Pr(Z \leq z)$ are in brackets

***, **, * Corresponds to the statistical significance at the 1, 5 and 10 % levels, respectively

very similar in nature), with the null hypothesis being that female children have at least as much EO as male children. The gains made by females are immediately clear for all parental attainment groups with the exception of parents with some college education. Taken in concert with the stochastic dominance results, where it was found that by the 70s, female children had a greater likelihood of achieving higher attainment levels than their male counterparts from the same cohort, the evidence here accords with empirical evidence of the narrowing in gender wage gap within the labour market, and the overall increased liberation of females from their circumstance relative to males.

3.3 The Generational Regression Approach

As noted in the introduction, a qualified equal opportunity program creates nonlinear effects for differing genders and parental attainment groups, which in turn has direct implications for regression analysis. To illustrate these effects here in the context of generational regressions, the model considered is of the form:

$$y_{i,k} = \beta_{0,k} + \beta_{1,k} x_{i,k} + \beta_{2,k} x^2_{i,k} + \epsilon_{i,k} \tag{12}$$

where $\mathbf{E}(\epsilon_{i,k}) = 0$ and $\mathbf{E}(\ln \epsilon^2_{i,k}) = \alpha_{0,k} + \alpha_{i,1} x_{i,k}$ where $i = \{1, 2, \ldots, n_k\}, k = \{male, female\}$. As before y corresponds to the child, and x the parent's outcome (in terms of educational attainment), and heteroskedasticity is modeled in terms of the log squared error being a linear function of parental attainment. That is a secondary regression will be performed on the residuals to examine the changes in variance across the cohorts,

$$\epsilon_{i,k} = \alpha_{0,k} + \alpha_{1,k} x_{i,k} + \eta_{i,k} \tag{13}$$

Note that parent and child variables here are both discrete integer variables which preclude a quantile regression approach, and would usually require some sort of multinomial technique for analysis since the residuals from regressions which employ them will have heteroskedastic errors. However the hypotheses considered here are that the regression relationship will become increasingly "convexified" over successive cohort regressions, and that heteroskedasticity will become increasingly negatively related to parental status, both of which for simplicity can readily be examined via the characteristics of a simple regression model with these albeit discrete variables. The results are reported in Table 7.

Table 7 Mobility OLS and an examination of heteroskedasticty by cohort

	1970s cohort		1950s cohort		≤1930s cohort	
	Father	Mother	Father	Mother	Father	Mother
	Panel A: Education of male child					
Parent's educ.	0.254*	0.049	0.413**	0.614***	0.925***	1.292***
	(0.152)	(0.163)	(0.183)	(0.173)	(0.305)	(0.300)
Parent's educ.^{2}	0.000	0.025	−0.029	−0.068**	−0.085	−0.170***
	(0.025)	(0.026)	(0.032)	(0.030)	(0.055)	(0.055)
Intercept	3.085***	3.440***	2.903***	2.727***	1.852***	1.530***
	(0.196)	(0.217)	(0.202)	(0.204)	(0.312)	(0.320)
R^2	0.128	0.084	0.075	0.063	0.108	0.096
σ^2	1.340	1.408	1.776	1.799	2.366	2.399
Nobs	895		995		569	
	Dependent variable: $\log(\epsilon^2)$					
Parent's educ.	−0.208***	−0.211***	−0.176***	−0.183***	−0.207***	−0.098*
	(0.043)	(0.045)	(0.044)	(0.046)	(0.053)	(0.055)
Intercept	−0.178	−0.080	0.040	0.117	0.599***	0.426***
	(0.135)	(0.137)	(0.102)	(0.102)	(0.098)	(0.105)
	Panel B: Education of female child					
Parent's educ.	0.508***	0.494***	−0.010	0.658***	1.246***	1.028***
	(0.123)	(0.126)	(0.148)	(0.145)	(0.228)	(0.229)
Parent's educ.^{2}	−0.049**	−0.044**	0.043*	−0.071**	−0.160***	−0.129***
	(0.021)	(0.021)	(0.026)	(0.026)	(0.041)	(0.041)
Intercept	3.021***	3.044***	3.415***	2.740***	1.257***	1.472***
	(0.156)	(0.162)	(0.169)	(0.167)	(0.234)	(0.237)
R^2	0.107	0.108	0.080	0.091	0.136	0.113
$\hat{\sigma}^2$	1.249	1.248	1.494	1.477	1.938	1.990
Nobs	1187		1201		887	
	Dependent variable: $\log(\epsilon^2)$					
Parent's educ.	−0.114***	−0.168***	−0.248***	−0.372***	0.017	−0.001
	(0.038)	(0.038)	(0.032)	(0.042)	(0.060)	(0.053)
Intercept	−0.587***	−0.405***	0.101	0.195**	−0.205**	−0.055
	(0.116)	(0.111)	(0.074)	(0.093)	(0.116)	(0.102)

Nine Provincial Indicators were included in each main regression

Standard errors are in parentheses

***, **, * Corresponds to the statistical significance at the 1, 5 and 10 % levels, respectively

At the outset, observe the positive effect that parental attainment has on child outcomes, regardless of gender and cohort, from both panels A and B of Table 7, with the effect waning more for male children than female. Adopting the usual interpretation that proximity of this coefficient to zero reveals proximity to independence, this suggests that male children made greater gains in mobility. This thus runs counter to the above findings utilizing the overlap measure, where it was found that gains in EO were stronger for females than they were for males. This suggests limitations in intergenerational mobility regressions relative to examinations of joint density functions, since the former approach

Table 8 Standard normal tests of OLS and heteroskedasticity results

	Panel A: Test of reduction in the degree of concavity in successive generations				Panel B: Test of increase in the degree of negative heteroskedasticity in successive cohorts			
	Male		Female		Male		Female	
	Father	Mother	Father	Mother	Father	Mother	Father	Mother
70s–50s	0.732	2.315***	−2.790	0.822	−0.519	−0.444	2.689	3.598
	[0.768]	[0.990]	[0.003]	[0.794]	[0.302]	[0.328]	[0.996]	[1.000]
70s–30s	1.425*	3.198***	2.438***	1.831**	−0.010	−1.596*	−1.841**	−2.549***
	[0.923]	[0.999]	[0.993]	[0.966]	[0.496]	[0.055]	[0.033]	[0.005]
50s–30s	0.891	1.632*	4.226***	1.180	0.456	−1.186	−3.911***	−5.483***
	[0.813]	[0.949]	[1.000]	[0.881]	[0.676]	[0.118]	[0.000]	[0.000]

$\Pr(Z \leq z)$ are reported in brackets

Test of reduction in concavity tests if the concavity coefficient in the later cohort is greater than in the earlier cohort. Test of increase in negative heteroskedasticity tests if the parental attainment coefficient for the later cohort is less than that of the earlier cohort

***, **, * Corresponds to the statistical significance at the 1, 5 and 10 % levels, respectively imposes functional restrictions as opposed to the latter approach which is nonparametric, and the effects are averaged across the socioeconomic/attainment groups. Next, note that the generational transfer technology appears to be concave as expected, exhibiting diminishing returns to parental ability. However, this concavity similarly diminished through the cohorts for both genders, highlighting the convexification of the relationship that QEO policies would engender.

Next, examining the coefficient for heteroskedasticity by gender, note that all the coefficients are negative and statistically significant with the exception of female children in the 30s cohort, affirming the prediction of the model that variances should be decreasing across socioeconomic/educational attainment groups. In addition, for both genders, the maternal effect was stronger amongst cohorts after the 30s. There also appears to be differential in the degree of heteroskedasticity, peaking for female children by the 50s cohort, but the 70s for males. This corresponds with the changes in EO observed using the overlap measure, where tendency towards EO slowed down more for females than they did for males (noting that female children still registered greater proximity to independence than for male children). These cross cohort changes in concavity of the parent–child transmission structure, and heteroskedasticity are tested in Table 8.

What becomes apparent from Table 8 for both reduction in concavity of the parent–child transmission structure, and the degree of heteroskedasticity, is that most of the significant changes occurred amongst female children, indicating that the beneficiaries of the EO policy were primarily female children. These gains in mobility, mirroring the analysis based on the stochastic dominance test, and overlap measure, peaked amongst the 50s cohort. Indeed, there is some evidence of significant increase in concavity amongst females between the 70s and 50s cohorts, supported by evidence of significant decrease in negative heteroskedasticity. Nonetheless, the primary point regarding a qualified equal opportunity program remains, that such a policy will not impinge on the progress of outcomes of the well endowed, be they male children, or children of higher socioeconomic/attainment parents.

4 Conclusions

It has been demonstrated that in the absence of sufficient flexibility or capacity in a society, the unqualified pursuit of an equal opportunity goal results in some of the inheriting generation being made worse off in absolute terms, while others are made better off relative to the status quo. If some sort of Pareto goal (in effect that no circumstance class should be made worse off) is also an objective of the policy maker, a qualified equal opportunity outcome emerges in which the most disadvantaged are addressed first. With such a program, complete independence of outcome from circumstance will not be observed across all socioeconomic groups, and conventional measures of mobility will not record complete mobility. Further, such policies have predictable consequences for generational regressions, thus suggesting ways that mobility measures could be re-interpreted. Evaluating conditional mobility policies via the transition matrix or joint distribution of outcomes, and circumstance requires indices which identify changes in mobility by subgroup, or conditional mobility measurement. In the context of generational regressions, qualified equal opportunity policies induce a reduction in concavity in the prevailing regression relationship, while also inducing heteroskedasticity in the corresponding error process, which is negatively related to the conditioning variable.

To illustrate the concept and the associated indices, the broad successes of various equal opportunity policies pursued either implicitly or explicitly in the emancipation of women, together with broad educational policies that affect all children were evaluated within the Canadian context. Specifically, the manner in which the gender gap in educational attainment narrowed, and how each succeeding cohort of children had a higher chance of obtaining a better educational outcome were examined. The qualified equal opportunity hypotheses received significant support from both the nonparametric stochastic dominance tests, and overlap measure in both cross gender, and cohort comparisons. Significant gains in mobility were observed through the decades, mainly amongst children of parents with high school, some college and technical education. Between the genders, female children were the key beneficiaries to the extent that by the 70s, the outcomes of female children dominated that of their male counterparts. Hypotheses relating to generational regressions that are consistent with a qualified equal opportunity program are not rejected for female children, but are weaker for males. All of which is what would have been expected from a qualified equal opportunity, or conditional mobility policy. It also appears that there is a segment of children, both males and females, of parents with little to no education whom society have neglected in that their mobility has diminished. It is conjectured that qualified policies that reflect the dominant needs of the modal populace may not be sufficiently broad in its reach to benefit the least endowed in society.

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